



In this worksheet students will practise proving and using fundamental trigonometric identities to simplify and solve equations. All questions relate solely to trigonometric identities and avoid topics from later units.

Easy Questions

1. Prove that $\sin^2 \theta + \cos^2 \theta = 1$.
2. Prove that $\sec \theta = \frac{1}{\cos \theta}$.
3. Prove that $\cot \theta = \frac{\cos \theta}{\sin \theta}$.
4. Prove that $1 + \tan^2 \theta = \sec^2 \theta$.
5. Verify that $\sin^2 \theta - \cos^2 \theta = 1 - 2 \cos^2 \theta$.

Intermediate Questions

6. Using $\sin^2 \theta + \cos^2 \theta = 1$, express $\sin^2 \theta$ in terms of $\cos^2 \theta$.
7. Prove that $\tan \theta = \frac{\sin \theta}{\cos \theta}$.
8. Prove the double-angle identity $\sin 2\theta = 2 \sin \theta \cos \theta$.
9. Prove that $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.
10. Show that $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$.
11. Show that $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$.
12. Simplify $\frac{1 - \cos 2\theta}{2}$ and explain its relation to $\sin^2 \theta$.
13. Prove that $1 - \tan^2 \theta = \frac{\cos 2\theta}{\cos^2 \theta}$.
14. Prove that $(1 - \sin \theta)(1 + \sin \theta) = \cos^2 \theta$.
15. Prove that $\frac{1 + \cos \theta}{\sin \theta} = \csc \theta + \cot \theta$.

16. Prove that $\frac{1 - \cos \theta}{\sin \theta} = \csc \theta - \cot \theta$.
17. Prove that $1 + \cot^2 \theta = \csc^2 \theta$.
18. Simplify the expression $\frac{1 - \cos^2 \theta}{\tan^2 \theta}$.
19. Prove that $\tan \theta + \cot \theta = \sec \theta \csc \theta$.

Hard Questions

19. Prove that $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$.
20. Prove that $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$.
21. Prove that $(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$.
22. Prove that $\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$.
23. Prove that $(1 + \sin \theta - \cos \theta)(1 + \sin \theta + \cos \theta) = 1 + 2 \sin \theta - \cos 2\theta$.
24. Prove that $(\tan \theta + \cot \theta)^2 = \sec^2 \theta \csc^2 \theta$.
25. Prove that $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2 \theta$.
26. Prove that $(\sin \theta - \cos \theta)^2 = 1 - 2 \sin \theta \cos \theta$.
27. Prove that $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$.
28. Prove that $\sin^2 \theta \cos^2 \theta = \frac{1 - \cos 4\theta}{8}$.
29. Prove that $(\sin \theta + \cos \theta)^4 - (\sin \theta - \cos \theta)^4 = 8 \sin \theta \cos \theta$.