ILLAWARRA TUTORING

In this worksheet you will learn how to calculate the probability of an event given that another event has already occurred. Work through the questions and show all your workings.

Easy Questions

1. Write the definition of conditional probability and use it to calculate $P(A|B)$ if $P(B) = 0.5$ and $P(A \text{ and } B) = 0.2$.

2. Suppose $P(A) = 0.4$ and $P(B) = 0.5$. If A and B are independent events, calculate $P(A|B)$ and explain why it equals $P(A)$.

3. A box contains several balls. If the probability of drawing a green ball is 0.3 and the probability of drawing a green ball that is also small is 0.12, what is the probability that a ball is small given it is green?

4. In a survey, 40% of respondents use a particular app and 15% use it daily. Calculate the probability that a respondent uses it daily given that they use the app.

5. If $P(\text{rain and traffic jam}) = 0.1$ and $P(\text{traffic jam}) = 0.25$, find $P(\text{rain} \,|\, \text{traffic jam})$.

Intermediate Questions

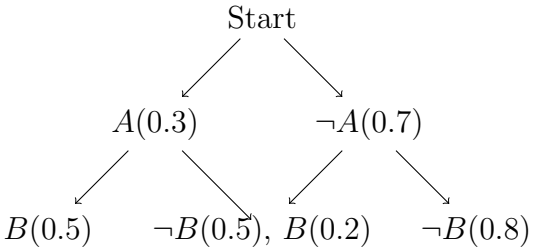
6. A bag contains 3 red balls and 4 blue balls. One ball is drawn and not replaced, and then a second ball is drawn. Find the probability that the second ball is red given that the first ball drawn was blue.

7. Let $P(A) = 0.35$ and $P(B) = 0.6$. If A and B are independent, verify that $P(A|B) = 0.35$.

8. In a survey, 60% of students like mathematics, 40% like science, and 25% like both. Calculate the probability that a student likes science given that they like mathematics.

9. A fair dice is rolled. Define event A as obtaining an even number and event B as obtaining a number greater than 3. Find $P(A|B)$.

10. The following tree diagram represents two successive events:



Using the diagram, calculate $P(B|A)$.

11. At a school, 30% of students play soccer, 20% play cricket, and 10% play both. Find the probability that a student plays cricket given that they play soccer.

12. In a standard deck of cards, what is the probability that a card is a heart given that it is red?

13. A box contains 5 white and 3 black balls. A ball is drawn, replaced, and then another ball is drawn. Find the probability that the second ball is white given that the first ball drawn was white.

14. A batch contains 100 items, of which 5 are defective. If an item is selected at random without replacement and is found to be defective, compute the probability that a second item selected from the remaining is defective.

15. An urn contains 8 balls, 3 of which are red. Two balls are drawn without replacement. Find the probability that the second ball is red given that the first ball drawn was red.

16. A jar contains 2 green, 3 yellow, and 5 blue marbles. A marble is drawn without replacement and it is known that the marble is not blue. Calculate the probability that the marble drawn is green on the second draw.

17. In a medical test, assume that 5% of the population has a disease. The test correctly identifies a diseased patient with probability 0.9 and falsely identifies a healthy patient as positive with probability 0.1. If a patient tests positive, compute the probability that they actually have the disease.

18. Prove that if events A and B are independent then $P(A|B) = P(A)$.

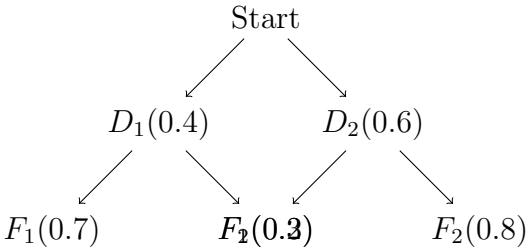
19. In a survey, 80% of people own a smartphone, 50% own a tablet, and 30% own both. Determine the probability that a person owns a smartphone given that they own a tablet.

20. Given that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \text{ and } B) = 0.3$, find $P(A|B)$.

Hard Questions

21. A factory has three production lines. Line 1 produces 30% of items with a defect rate of 2%, Line 2 produces 50% with a defect rate of 3%, and Line 3 produces the remaining 20% with a defect rate of 5%. If an item is found to be defective, calculate the probability that it was produced by Line 1.

22. The following tree diagram shows the outcomes of a two-stage process:



Use the diagram to find $P(D_1|F_1)$.

23. Prove that for any events A and B with $P(B) > 0$, it holds that $P(A|B) = 1 - P(\neg A|B)$.

24. In a standard deck of 52 cards, two cards are drawn without replacement. Given that the first card drawn is an ace, determine the probability that the second card is a heart.

25. If $P(A|B) = 0.8$ and $P(B) = 0.25$, calculate $P(A \text{ and } B)$.

26. In a classroom, 40% of students passed mathematics, 50% passed science and 20% passed both subjects. Find the probability that a student passed mathematics provided they passed science.

27. In a delivery service, 30% of orders are delivered late and among these, 10% are damaged. What is the probability that an order is damaged given that it was delivered late?

28. Given that $P(Y|X) = 0.5$, $P(Y|\neg X) = 0.2$ and $P(X) = 0.4$, compute $P(Y)$.

29. In a game, the probability of winning on a turn given that the previous turn was a win is 0.4, and if the previous turn was a loss the probability of winning the next turn is 0.2. If the first turn is a win, calculate the probability of winning on the third turn.

30. Events A, B and C have the following probabilities: $P(A) = 0.5$, $P(B) = 0.4$, $P(C) = 0.3$; $P(A \text{ and } B) = 0.2$, $P(A \text{ and } C) = 0.15$, $P(B \text{ and } C) = 0.12$ and $P(A \text{ and } B \text{ and } C) = 0.08$. Compute $P(A \,|\, B \text{ and } C)$.