

In this worksheet you will learn to calculate the second derivative to analyse the concavity of functions and identify points of inflection. You will apply these skills to a variety of functions and explain the changes in concavity.

Easy Questions

- 1. The function is given as $f(x) = x^2$. Calculate the first derivative and then the second derivative of the function.
- 2. The function is defined as $f(x) = x^3$. Determine the first and the second derivatives of the function.
- 3. For the function f(x) = 2x + 1, compute the first derivative and then the second derivative.
- 4. Given f(x) = 7, find the second derivative and explain your result.
- 5. For $f(x) = x^4$, compute the second derivative and state its sign at x = 0.

Intermediate Questions

- 6. Consider $f(x) = x^3 3x$. Find the second derivative and determine the intervals where the function is concave up or concave down.
- 7. For the function $f(x) = x^4 4x^2$, compute its second derivative and identify the intervals of concavity as well as any possible points of inflection.
- 8. The function is given by $f(x) = \frac{1}{3}x^3 x$. Calculate its second derivative and identify the point(s) of inflection.
- 9. Given $f(x) = (x 1)^2$, compute its second derivative and discuss the concavity of the function.
- 10. For $f(x) = x^3 + x^2 x$, determine the second derivative and then identify any points where the concavity might change.
- 11. Consider $f(x) = x^5$. Compute the second derivative and indicate the intervals where the function is concave up and concave down.
- 12. For $f(x) = x^3 6x^2 + 9x$, compute the second derivative and discuss the concavity of the function.

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- 13. The function is $f(x) = -x^2 + 4x$. Find the first and second derivatives and then describe the concavity.
- 14. Given $f(x) = 3x^2 12x + 5$, calculate the second derivative and explain the overall concavity of the graph.
- 15. For $f(x) = x^4 + 2x^3 8x$, find the second derivative and determine the possible points of inflection, justifying your answer.
- 16. Consider $f(x) = x^3 3x^2 + 2$. Compute the second derivative and state the intervals where the graph is concave up and concave down.
- 17. For $f(x) = 2x^2 + 3x + 1$, find the second derivative and evaluate it for all x to determine the concavity.
- 18. Given $f(x) = x^4$, compute the second derivative at x = 0 and provide an explanation of the result.
- 19. For $f(x) = -x^3$, determine the second derivative and calculate its value at x = -2.
- 20. The function is $f(x) = \frac{x^2 4}{2}$. Calculate the second derivative and determine the intervals of concavity.

Hard Questions

- 21. For $f(x) = x^4 4x^3 + 6x^2 4x + 1$, compute f''(x) and determine all points where the concavity changes.
- 22. Given $f(x) = x^5 5x^3 + 4x$, calculate the second derivative and find all potential points of inflection.
- 23. Consider $f(x) = x^3 6x^2 + 9x + 1$. Compute the second derivative and, using the result, sketch a diagram to illustrate the regions where the function is concave up and concave down.
- 24. For $f(x) = x^4 2x^2$, calculate f''(x) and find the points of inflection. Explain how the concavity of the function changes at these points.
- 25. The function is $f(x) = -2x^3 + 9x^2 12x + 1$. Compute the second derivative and use your result to discuss the intervals where the graph is concave up or down.
- 26. Given $f(x) = x^6 3x^4$, determine the second derivative and identify any critical points related to changes in concavity.
- 27. Consider $f(x) = -x^4 + 4x^3 6x^2$. Calculate the second derivative and specify the intervals where the function is concave upward.
- 28. For $f(x) = x^5 + x^2 5x + 3$, determine the second derivative and discuss any potential points of inflection.

- 29. Given $f(x) = x^4 4x$, compute the second derivative and, using your results, sketch (using pen and paper) a diagram of the graph emphasising the regions of concavity. Provide a clear explanation of your process.
- 30. For $f(x) = 2x^3 9x^2 + 12x 4$, compute the second derivative and identify all points of inflection. Provide a full justification for how you determined the points.

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