

In this worksheet you will investigate how reflecting a function over an axis changes its graph, enhancing your understanding of symmetry. You will explore reflections over the x-axis and y-axis, compute coordinates after reflections, and draw graphs to visualise the transformations.

## Easy Questions

- 1. Explain in plain text what reflecting a function f(x) over the x-axis does to its output.
- 2. Explain in plain text what reflecting a function f(x) over the y-axis does to its input.
- 3. For the function f(x) = x + 2, write the equation of the function that represents its reflection in the x-axis.
- 4. For the function  $f(x) = x^2$ , write the equation of the function after reflecting over the y-axis.
- 5. Using the function f(x) = x, sketch the graph of f(x) and also sketch its reflection over the x-axis on the same coordinate axes.

## Intermediate Questions

- 6. Given the point (3, 4), find its reflection over the x-axis.
- 7. Given the point (3, 4), find its reflection over the y-axis.
- 8. For the function f(x) = 2x 3, graph both f(x) and its reflection over the x-axis on the same set of axes.
- 9. The function  $f(x) = \sqrt{x}$  is defined for  $x \ge 0$ . Write down the equation of its reflection over the y-axis and describe its domain.
- 10. For the function  $f(x) = x^2$ , write the expressions for f(-x) and -f(x). State if the reflections over the y-axis and x-axis yield the same graph, and explain why or why not.
- 11. For the function f(x) = |x|, compute f(-x) and -f(x). Explain what each transformation represents graphically.

- 12. The function  $f(x) = \frac{1}{x+1}$  is given. Write the expression for its reflection over the y-axis.
- 13. Explain why reflecting the function  $f(x) = x^2$  over the y-axis results in an identical graph.
- 14. Describe in words how reflecting the function  $f(x) = x^3$  over the x-axis affects its graph.
- 15. If the point (-2, 5) lies on the graph of f(x), determine its coordinates after reflecting over the x-axis and over the y-axis.
- 16. Graph the function  $f(x) = x^2$  and its reflection over the x-axis  $-x^2$ . Identify the main differences in their graphs.
- 17. For the function  $f(x) = \sin x$ , write the equations for its reflection over the x-axis and its reflection over the y-axis.
- 18. Determine whether reflecting the function  $f(x) = \cos x$  over the y-axis produces a graph that is identical to the original function. Explain your reasoning.
- 19. Show that reflecting a function over the x-axis and then the y-axis is equivalent to rotating its graph by 180 degrees. Use the function  $f(x) = e^x$  to illustrate your answer.
- 20. For the function  $f(x) = x^2 4$ , write the equation of its reflection over the x-axis.

## Hard Questions

- 21. Prove algebraically that reflecting a function f(x) over the y-axis gives f(-x), and that reflecting over the x-axis gives -f(x). Provide detailed justification in your answer.
- 22. Given  $f(x) = (x-2)^2 + 3$ , find the equation of the function after reflecting it over the x-axis. Then determine the coordinates of the vertex of the reflected function.
- 23. Determine the equation for the reflection over the y-axis of the function  $f(x) = \frac{1}{x-1}$  (with  $x \neq 1$ ). Then sketch the graph of the reflected function.
- 24. The function  $f(x) = \sqrt{x+4}$  has a specific range. Find the equation of its reflection over the x-axis and state the new range.
- 25. Given  $f(x) = x^3 3x$ , find and compare the functions f(-x) and -f(x). Discuss the differences in symmetry between these two reflections.
- 26. Provide an algebraic argument or a counterexample to show that reflecting a function over the y-axis does not always yield the same graph as reflecting it over the x-axis.
- 27. For the function f(x) = |x 2| + 1, determine the equation of the function after reflecting it over the y-axis. Comment on how the vertex of the graph is affected.

- 28. Consider the function  $f(x) = \frac{2}{x}$ , where  $x \neq 0$ . Determine the equations of the functions resulting from reflecting f(x) over the x-axis, over the y-axis, and over both axes. Discuss how the asymptotes change with each reflection.
- 29. For the function  $f(x) = \ln(x+2)$ , find the equation of its reflection over the y-axis. State the new domain of the function and describe the asymptotes.
- 30. Consider the function  $f(x) = \sin x$  for  $x \in [-\pi, \pi]$ . Determine the equations for the reflection of its graph across the x-axis and across the y-axis. Then, describe in detail the symmetry properties of the resulting functions.