ILLAWARRA TUTORING

In this worksheet you will learn to define discrete random variables and understand how their distributions work. A discrete random variable assigns a numerical value to each outcome of an experiment and its distribution (or probability mass function) lists the probabilities for these values.

Easy Questions

1. Define a discrete random variable in your own words.
2. Provide an example of a discrete random variable that could arise from tossing a coin once.
3. For a fair coin toss, list the sample space and explain why it is discrete.
4. Consider flipping two fair coins. State whether the number of heads is a discrete random variable and justify your answer.
5. State the key property that every discrete probability distribution must satisfy regarding the sum of probabilities of all outcomes.

Intermediate Questions

6. A standard six-faced die is rolled and the outcome is recorded as X, where X is the number shown. List the sample space for X and describe briefly what the probability distribution (pmf) would look like in this case.
7. Construct a probability mass function table for the random variable X defined in Question 6.

x	$P(X = x)$
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

8. A spinner is divided into 4 equal regions numbered 1 to 4. Define the random variable Y to be the number on which the spinner lands. List the probability distribution for Y.

9. Explain why a random variable must assign a numerical value to each outcome in an experiment rather than a non-numerical description.

10. Describe briefly the difference between a discrete random variable and a continuous random variable.

11. A bag contains 3 red balls and 2 blue balls. Define a random variable Z such that $Z = 1$ if a red ball is drawn and $Z = 0$ if a blue ball is drawn. List the possible values of Z and their probabilities.

12. Using the random variable defined in Question 11, verify that the total probability of all outcomes is 1.

13. Let X be the sum obtained by rolling two dice once. Is X a discrete random variable? Provide a brief explanation.

14. How can the probability mass function (pmf) of a discrete random variable be represented?

15. Formulate a scenario in which a discrete random variable takes exactly three distinct numerical values. Describe the scenario and list the possible values.

16. Consider the following table:

x	$P(X = x)$
1	0.2
2	0.5
3	0.3

Determine if this represents a valid probability distribution. Explain your reasoning.

17. Define the cumulative distribution function (CDF) for a discrete random variable in plain terms.

18. List two important properties that any probability mass function must satisfy.

19. Consider a box containing 10 numbered cards (from 1 to 10). Define a random variable W that equals the card number if the card is odd, and 0 if the card is even. List the possible values of W and their probabilities, assuming each card is equally likely.

20. Describe a real-life scenario where a discrete random variable naturally arises. Identify the variable, list its possible outcomes, and explain how you would assign probabilities to these outcomes.

Hard Questions

21. Consider the experiment of flipping three coins and letting H denote the number of heads obtained. Construct the probability mass function table for H and justify why the table is valid.

22. A standard die is rolled three times. Define the random variable X as the difference between the highest and lowest values observed. List all possible values of X and describe qualitatively how its distribution might look.

23. Suppose a random variable Y is defined as the square of the number shown on a fair die roll. Derive the probability distribution for Y.

24. An experiment involves drawing two cards without replacement from a small set of cards numbered $1, 2, 3, 4$. Define the random variable R as the sum of the two drawn cards. List all possible values for R and describe, in words, how you would determine the probability associated with each sum.

25. Prove that for any discrete random variable, the sum of the probabilities across all possible outcomes equals 1, using the axioms of probability.

26. Suppose a random variable X takes values $1, 3, 5$ with probabilities p_1, p_2, p_3, respectively. List the constraints on p_1, p_2, p_3 for this to be a valid probability distribution.

27. Devise your own scenario in a real-life context where you can define a discrete random variable. Describe the scenario, specify the random variable with its possible outcomes and assign appropriate probabilities.

28. Consider two different discrete random variables defined on distinct sample spaces. Explain how their probability distributions can be compared and what information each distribution provides about the underlying experiment.

29. A shooter continues to fire at a target until they hit it. Define a discrete random variable T as the number of attempts needed. Outline the steps you would take to theoretically build the probability distribution of T.

30. Imagine a special six-faced die where the faces are not uniformly distributed. Suppose the probability for face i is given by $P(i) = \dfrac{w_i}{\sum_{j=1}^{6} w_j}$, where w_i are positive weights. Propose a set of weights (for instance, $w_1 = 1, w_2 = 2, w_3 = 3, w_4 = 4, w_5 = 5, w_6 = 6$), calculate the corresponding probabilities, and verify that their sum is 1.